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On unquenched $\mathcal{N} = 2$ holographic flavor

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Abstract: The addition of fundamental degrees of freedom to a theory which is dual (at low energies) to $\mathcal{N} = 2$ SYM in 1+3 dimensions is studied. The gauge theory lives on a stack of N_c D5 branes wrapping an S^2 with the appropriate twist, while the fundamental hypermultiplets are introduced by adding a different set of N_f D5-branes. In a simple case, a system of first order equations taking into account the backreaction of the $N_f \sim N_c$ flavor branes is derived. From it, the modification of the holomorphic coupling is computed explicitly. Mesonic excitations are also discussed.

Keywords: Brane Dynamics in Gauge Theories, Gauge-gravity correspondence.

Contents

1. Introduction and summary of results

In its original and most studied form [1], the gauge-string correspondence relates a field theory where all fields transform in the adjoint representation of the gauge group to a theory of closed strings. An enormous amount of work has been devoted to generalize the correspondence in different directions. Since the physics of real world strong interactions is dominated by mesons and baryons, a generalization of obvious importance is to include matter transforming in the fundamental representation of the gauge group. Early works on the subject were [2–4].

In the seminal papers [5, 6], it was argued that including a small number of flavors (N_f fixed, $N_c \to \infty$) corresponds in the dual theory to adding an open string sector due to the presence of a few flavor branes. In the field theory, this is a quenched approximation in the sense that the diagrams with fundamentals running in the loops are suppressed by N_c^{-1}. In the gravity side, one can neglect the backreaction of the branes on the geometry. These ideas and methods sparked a lot of activity which led to a neat understanding of many different phenomena in such a limit, in a variety of setups.

Nevertheless, field theories in which N_f is of the same order as N_c are amazingly rich as is clear, for instance, from Seiberg's analysis of $\mathcal{N}=1$ SQCD [7]. In addition, the fact

JHEP12(2006)032

that in the real world $\frac{N_f}{N_c}$ is of order one is reflected in experimental facts as, among others, multihadron production or the large mass of the η' meson. This motivates the study of string duals in the Veneziano limit $N_c \to \infty$ with $g^2 N_c$ and $\frac{N_f}{N_c}$ fixed. In order to maintain the curvature small, strong coupling (large $g^2 N_c$) is required. However, this case is not as well understood as the quenched one. Some works along this direction studying four-dimensional field theories are [2, 3, 8 − 12] (three-dimensional field theories were addressed in [13 − 15]). In one way or another, all these models have a region of large curvature associated to the presence of the fundamental quarks.

In this note, a setup consisting of wrapped branes, dual to $\mathcal{N} = 2$ SQCD is presented. By appropriately smearing the flavor branes such that the shell of branes forms a domain wall in the geometry, they do not generate any pathological region in the geometry (as long as $N_f \leq 2N_c$ so there is no Landau pole in the dual field theory). This smearing corresponds to a smearing of the complex masses of the hypermultiplets, more details are given in section 3.2. Anyhow, the solution has a naked curvature singularity in the IR and a divergent dilaton in the UV. These features are already present in the original setup and are not related to the presence of flavor branes. Some comments on how these pathologies can be dealt with can be found in sections 2 and 4 respectively.

The framework is the gravity dual of $\mathcal{N} = 2$ SYM introduced in [16, 17]. It consists of D5-branes wrapping an S^2 inside a CY_2 in such a way that eight supercharges are preserved. At small energies compared to the inverse radius of the sphere, the theory becomes effectively four-dimensional. It will be shown how one can add a different kind of D5-branes in a way that no further supersymmetry is broken. They introduce an open string sector that corresponds to having in the field theory (massive) hypermultiplets transforming in the fundamental representation of the gauge group. As explained above, when the number of these branes is comparable to the number of colors, their backreaction on the geometry has to be taken into account. This is done by applying the procedure advocated in [12]: the D5-branes are sources for the Ramond-Ramond $F_{(3)}$ field strength, and therefore they modify its Bianchi identity. Once this is taken into account, the set of first order equations can be found by imposing the vanishing of the supersymmetry variation of the fermion fields. The resulting system of equations is quite complicated and I have not managed to find an analytical solution. However, as it will be shown, it can indeed be explicitly solved in a certain region of the space. This is enough to prove that the modification of the couplings due to the flavor branes is the one expected from field theory. Finally, the meson spectrum is studied. The excitations can be rearranged in a tower of massive $\mathcal{N} = 2$ vector multiplets. Due to the UV pathological behavior of the solution, a regularization procedure is needed in order to estimate the masses of the physical excitations. Once this procedure is implemented, the dependence of the meson masses on the different physical parameters is briefly discussed.

Apart from the interest of the particular model that will be addressed, one of the aims of this paper is to make clearer the methods of [12] by their application to a simpler setup. In particular, as stated above, in this $\mathcal{N} = 2$ construction the flavor branes do not spark the divergence or vanishing of any component of the metric. Hopefully, this note

JHEP12(2006)032

can contribute in the development of techniques for the study of holographic duals with unquenched matter.

The contents of the paper are organized as follows: in section 2, the unflavored solution is reviewed and rewritten with a notation that will be useful in the following. In section 3.1, the embedding of the flavor branes is discussed and the physical meaning of the different spacetime directions is explained. In section 3.2, the system of equations including the backreaction is obtained and in section 3.3 it is shown how the couplings are modified by the unquenched flavor. Section 4 deals with the spectrum of excitations. A few WKB formulae used in section 4.4 are summarized in appendix A.

2. A review of the dual of $\mathcal{N}=2$ SYM

In references [16, 17], a IIB gravity dual of $\mathcal{N}=2$ SYM in the Coulomb branch was found. The ten-dimensional solution was obtained as an uplift from seven-dimensional SO(4) gauged supergravity. It corresponds to 5-branes[1] wrapping a two-sphere with the appropriate twisting to preserve eight supercharges, i.e. $\mathcal{N}=2$ in the effective four-dimensional low energy theory. Geometrically, it corresponds to wrapping the branes along a compact SLag two-cycle inside a Calabi-Yau two-fold. This leaves two flat transverse dimensions which are identified with the moduli space corresponding to giving vevs to the complex scalar inside the $\mathcal{N}=2$ vector multiplet.

The solution presented below has a curvature singularity in the IR. The singularity is good according to the criterion of [18]. In fact, it was shown in [19], by considering wrapped NS5-branes that the singularity is an artifact of the supergravity approximation and is resolved by the worldsheet CFT. Therefore, the singularity will not be a matter of concern in the following.

This section does not contain original material but reviews the results of [16, 17, 20] in order to fix notation for the following and make this note reasonably self-contained. An excellent review of this model and similar constructions is [21].

2.1 The solution

The metric (Einstein frame) of the solution found in [16, 17] reads:

$$ds_{10}^2 = N_c g_s \alpha' e^{\frac{\Phi}{2}} \left[\frac{1}{N_c g_s \alpha'} dx_{1,3}^2 + z(d\tilde{\theta}^2 + \sin^2\tilde{\theta} d\tilde{\varphi}^2) + e^{2x} dz^2 + d\theta^2 + \right.$$
$$\left. + \frac{e^{-x}}{\Omega} \cos^2\theta (d\phi_1 + \cos\theta d\tilde{\varphi})^2 + \frac{e^{x}}{\Omega} \sin^2\theta d\phi_2^2 \right] , \tag{2.1}$$

there is a magnetic RR twoform:

$$C_{(2)} = N_c g_s \alpha' \phi_2 d \left[\frac{\sin^2\theta}{\Omega e^x} (d\phi_1 + \cos\tilde{\theta} d\tilde{\varphi}) \right] \tag{2.2}$$

[1] Originally, the solution was built with NS5-branes. In the following, the S-dual solution corresponding to D5-branes will be considered.

and the dilaton is given by:

$$e^{2\Phi} = e^{2\Phi_0} e^{2z} \left[1 - \sin^2\theta \frac{1+ce^{-2z}}{2z}\right] \tag{2.3}$$

and we have defined the quantities:

$$\begin{aligned} e^{-2x} &= 1 - \frac{1+ce^{-2z}}{2z} \\ \Omega &= e^{x}\cos^2\theta + e^{-x}\sin^2\theta \end{aligned} \tag{2.4}$$

The angles take values in the following intervals: $\tilde{\theta} \in [0,\pi]$, $\theta \in [0, \frac{\pi}{2}]$, $\tilde{\varphi}, \phi_1, \phi_2 \in [0, 2\pi)$. The variable z ranges from a value z_0 for which $e^{-2x(z_0)} = 0$ up to infinity.

There are two integration constants on which the solution depends, Φ_0 and c. Following the discussion of [16, 17], the solution is dual to $\mathcal{N} = 2$ SYM at points of the Coulomb branch in which the vevs for the entries of the complex scalar matrix are distributed in a ring (they have fixed modulus whereas the phase is smeared). The constant $c \geq -1$ determines the size of the ring. Solutions with $c < -1$ are unphysical and, in fact, the singularity becomes of the bad type. Notice that although taking a single vev different from zero spontaneously breaks the $\mathrm{U}(1)_R$, this smearing procedure restores it, at least if one ignores N_c^{-1} effects, where one could start seeing that the distribution is not continuous (see figure 1 in section 3.2). Points of the Coulomb branch where the $\mathrm{U}(1)_R$ is not restored were discussed in [17].

The quantization condition is:

$$\frac{1}{2\kappa_{(10)}^2} \int_{S^3} F_{(3)} = N_c T_5 \tag{2.5}$$

where S^3 is the transverse three-sphere parameterized by θ, ϕ_1, ϕ_2 at $z \to \infty$, yielding:

$$\int_{S^3} F_{(3)} = N_c g_s \alpha' \int_0^{2\pi} d\phi_1 \int_0^{2\pi} d\phi_2 \int_0^{\frac{\pi}{2}} 2\sin\theta\cos\theta d\theta = 4\pi^2 N_c g_s \alpha' \tag{2.6}$$

Taking into account:

$$T_5 = \frac{1}{(2\pi)^5 g_s \alpha'^3}\,, \qquad \frac{1}{2\kappa_{(10)}^2} = \frac{1}{(2\pi)^7 g_s^2 \alpha'^4} \tag{2.7}$$

one can immediately check that (2.5) is satisfied.

2.2 Rewriting the solution and Killing spinors

In reference [20], the solution was rewritten in different variables which allow a better understanding of the physics. Define:

$$\rho = \sin\theta e^{z}\,, \qquad \sigma = \sqrt{z}\cos\theta e^{z-x}\,, \tag{2.8}$$

so the metric (2.1) reads:

$$\begin{aligned} ds_{10}^2 = g_s N_c \alpha' e^{\frac{\Phi}{2}} \bigg[& \frac{1}{g_s N_c \alpha'} dx_{1,3}^2 + z(d\tilde{\theta}^2 + \sin^2\tilde{\theta} d\tilde{\varphi}^2) + \\ & + e^{-2\Phi}(d\rho^2 + \rho^2 d\phi_2^2) + \frac{e^{-2\Phi}}{z}\Big(d\sigma^2 + \sigma^2(d\phi_1 + \cos\tilde{\theta} d\tilde{\varphi})^2\Big)\bigg] \end{aligned} \tag{2.9}$$

Written in this way, it is clear that the Calabi-Yau twofold directions are $\tilde{\theta}, \tilde{\varphi}, \sigma, \phi_1$ (of course, in this solution with fluxes there is not a Calabi-Yau any more, but it can be thought of as a deformation of the Calabi-Yau that was present before backreaction). The coordinates ρ, ϕ_2 span the transverse two-dimensional plane, so they should be identified with the moduli space, and therefore rotations in ϕ_2 are related to the $\mathrm{U}(1)_R$ symmetry of the field theory. These statements will be made more precise below.

Actually, one could start by writing the metric (2.9) as an ansatz with $z(\rho, \sigma)$, $\Phi(\rho, \sigma)$. The ansatz for the RR 3-form would be:

$$\begin{aligned} F_{(3)} = N_c g_s \alpha' \Big[& -g' d\phi_2 \wedge d\rho \wedge (d\phi_1 + \cos\tilde{\theta} d\tilde{\varphi}) - \dot{g} d\phi_2 \wedge d\sigma \wedge (d\phi_1 + \cos\tilde{\theta} d\tilde{\varphi}) + \\ & + g \sin\tilde{\theta} d\phi_2 \wedge d\tilde{\theta} \wedge d\tilde{\varphi} \Big] \end{aligned} \tag{2.10}$$

which ensures $dF_{(3)} = 0$ and we have defined:

$$' \equiv \partial_\rho , \qquad \dot{} \equiv \partial_\sigma \tag{2.11}$$

Introducing this ansatz in the IIB transformations for the fermions:

$$\begin{aligned} \delta\lambda &= \frac{1}{2} \partial_\mu \Phi \Gamma^\mu \sigma_1 \epsilon + \frac{1}{24} e^{\frac{\Phi}{2}} \Gamma^{\mu_1\mu_2\mu_3} \epsilon F_{\mu_1\mu_2\mu_3} = 0 , \\ \delta\psi_\mu &= \partial_\mu \epsilon + \frac{1}{4} \omega_\mu^{ab} \Gamma^{ab} \epsilon + \frac{1}{96} e^{\frac{\Phi}{2}} \left(\Gamma_\mu^{\ \mu_1\mu_2\mu_3} - 9 \delta_\mu^{\mu_1} \Gamma^{\mu_2\mu_3} \right) \sigma_1 \epsilon F_{\mu_1\mu_2\mu_3} = 0 , \end{aligned} \tag{2.12}$$

a set of first order equations can be obtained. Consider the orthonormal frame:

$$\begin{aligned} & e^i = e^{\frac{\Phi}{4}} dx_i \ (i = 0, \dots, 3) , \quad e^4 = \sqrt{g_s N_c \alpha'} e^{\frac{\Phi}{4}} \sqrt{z} d\tilde{\theta} , \quad e^5 = \sqrt{g_s N_c \alpha'} e^{\frac{\Phi}{4}} \sqrt{z} \sin\tilde{\theta} d\tilde{\varphi} , \\ & e^6 = \sqrt{g_s N_c \alpha'} e^{-\frac{3\Phi}{4}} d\rho , \quad e^7 = \sqrt{g_s N_c \alpha'} e^{-\frac{3\Phi}{4}} \rho d\phi_2 , \\ & e^8 = \sqrt{g_s N_c \alpha'} \frac{e^{-\frac{3\Phi}{4}}}{\sqrt{z}} d\sigma , \quad e^9 = \sqrt{g_s N_c \alpha'} \frac{e^{-\frac{3\Phi}{4}}}{\sqrt{z}} \sigma (d\phi_1 + \cos\tilde{\theta} d\tilde{\varphi}) \end{aligned} \tag{2.13}$$

The Killing spinors are given by:

$$\epsilon = e^{\frac{\Phi}{8}} e^{-\frac{1}{2}\phi_1 \Gamma_{45} + \frac{1}{2}\phi_2 \Gamma_{67}} \eta \tag{2.14}$$

where η is a constant spinor satisfying:

$$\sigma_1 \Gamma_{4567} \eta = -\eta , \qquad \Gamma_{4589} \eta = \eta \tag{2.15}$$

The first order equations read:

$$g = -\rho z' , \tag{2.16}$$
$$e^{2\Phi} = \frac{\sigma}{z \dot{z}} , \tag{2.17}$$
$$g' = -2 e^{-2\Phi} \rho \sigma \dot{\Phi} , \tag{2.18}$$
$$\dot{g} = -z^{-2} e^{-2\Phi} \sigma g + 2 z^{-1} \rho \sigma e^{-2\Phi} \Phi' . \tag{2.19}$$

It is possible to check that these equations ensure the equation of motion for the 3-form $d(e^\Phi * F_{(3)}) = 0$. Notice, however, that these equations are not independent since (2.19) is automatically implied by (2.16) and (2.17). The system (2.16)-(2.19) can be rephrased as a single second order non-linear partial differential equation (PDE) for the function $z(\rho,\sigma)$:

$$\rho z(\dot z - \sigma \ddot z) = \sigma(\rho \dot z^2 + z' + \rho z'') \tag{2.20}$$

and once z is known, g and Φ can be obtained from (2.16), (2.17). The implicit relation deduced from (2.8):

$$\sigma^2 - z e^{-2x}(e^{2z} - \rho^2) = 0 \tag{2.21}$$

(where e^{-2x} was defined in (2.4)) solves (2.20) and yields the known solution[2] (2.1)-(2.4). The value of g in terms of the old variables can be read by comparing (2.2) to (2.10):

$$g = -\frac{\sin^2\theta}{e^{2x}\cos^2\theta + \sin^2\theta} \tag{2.22}$$

It is also useful to define quantities z_0, ρ_0 which depend on c as:

$$e^{-2x(z_0)} = 0\ , \qquad \rho_0 = e^{z_0} \tag{2.23}$$

The minimum value that z can attain is z_0. When $c = -1$, then $z_0 = 0$.

Using (2.9) and (2.15), it is immediate to see that two kinds of D5-brane probes can be added to the setup preserving the full supersymmetry of the background. They correspond to the κ-symmetry projectors $\sigma_1\Gamma_{012345}$ and $\sigma_1\Gamma_{012389}$. The first kind corresponds to D5 extended along $x_0,\ldots,x_3,\tilde\theta,\tilde\varphi$ placed at $\sigma = 0$ (notice that the dilaton does not diverge at $\sigma = 0$ despite the appearance of (2.17) since $\dot z \sim \sigma$ near $\sigma = 0$ as can be deduced from (2.20)). These branes, which wrap a compact S^2, are the color branes. Computing the Born-Infeld action on their worldvolume leads, upon reduction to four dimensions, to the $\mathcal{N} = 2$ SYM action. In [20], the identification of the Yang-Mills coupling, the θ_{YM} and the radius energy relation was made precise. In order to follow their argument, notice that at $\sigma = 0$:

$$(\sigma = 0,\ \rho < \rho_0) \Rightarrow \begin{cases} z = z_0 \\ g = 0 \end{cases} ,\qquad (\sigma = 0,\ \rho > \rho_0) \Rightarrow \begin{cases} z = \log\rho \\ g = -1 \end{cases} , \tag{2.24}$$

where we have used (2.21), (2.16) in order to derive these expressions. $\sigma = 0,\ \rho \le \rho_0$ is the singular locus of the geometry. As acknowledged in [16], it is remarkable that one obtains regular results for physical quantities despite the singularity.

Now, consider the abelian worldvolume DBI+WZ action of a D5-brane probe placed at $\sigma = 0$. Then, expand it to leading order and promote it to a non-abelian one. Normalizing the SU(N_c) generators as $tr(T^A T^B) = \frac{1}{2}\delta^{AB}$, one obtains [20]:

$$S_{YM} = -\frac{1}{g_{YM}^2}\int d^4x \{\frac{1}{4}F^A_{\alpha\beta}F_A^{\alpha\beta} + \frac{1}{2}D_\alpha \bar\Psi^A D^\alpha \Psi_A\} + \frac{\theta_{YM}}{32\pi^2}\int d^4x F^A_{\alpha\beta}\tilde F_A^{\alpha\beta} \tag{2.25}$$

[2] Simple solutions of (2.20) are $z = c_1 + c_2\log\rho$ which is not physical since from (2.17) one gets $\Phi = \infty$ and $z = c_1\sqrt{2\sigma^2 + c_2}$ which leads to constant dilaton and $g = F_{(3)} = 0$ and just Ricci flat $\mathbb{R}_{1,5}\times EH_4$ where EH_4 denotes the Eguchi-Hanson space.

JHEP12(2006)032

where we have defined Ψ, which is the complex scalar of the $\mathcal{N}=2$ vector multiplet as:

$$\Psi = \frac{\sqrt{g_s N_c \alpha'}}{2\pi\alpha'} \rho e^{i\phi_2} \tag{2.26}$$

After integrating the S^2 parameterized by $\tilde{\theta}, \tilde{\varphi}$, the value of the couplings can be read from the solution (2.24). For $\rho < \rho_0$ they are constant while for $\rho > \rho_0$ they read:

$$\frac{1}{g_{YM}^2} = \frac{N_c}{4\pi^2}(z|_{\sigma=0}) = \frac{N_c}{4\pi^2}\log\rho\,, \tag{2.27}$$

$$\theta_{YM} = 2(g|_{\sigma=0})N_c\phi_2 = -2N_c\phi_2 \tag{2.28}$$

Now, noticing that the complex scalar Ψ has protected mass dimension one, one is lead, from (2.26) to the following radius-energy relation:

$$\rho = \frac{\mu}{\Lambda} \tag{2.29}$$

where μ is the mass scale at which the theory is defined and Λ the dynamically generated mass scale. Then:

$$\beta(g_{YM}) \equiv \frac{\partial g_{YM}}{\partial \log(\mu/\Lambda)} = \frac{\partial g_{YM}}{\partial \log\rho} = -\frac{N_c}{8\pi^2} g_{YM}^3 \tag{2.30}$$

On the other hand, one can compute the chiral anomaly, which is associated to $\mathrm{U}(1)_R$ shifts:

$$\phi_2 \to \phi_2 + 2\epsilon \quad \Rightarrow \quad \theta_{YM} \to \theta_{YM} - 4N_c\epsilon\,, \qquad \epsilon \in [0, 2\pi) \tag{2.31}$$

The parameter ϵ takes values in $[0, 2\pi)$ since, although in the geometry one comes back to the same point after a rotation of 2π in ϕ_2, the complete $\mathrm{U}(1)_R$ rotation is of a 4π angle. This can be seen from (2.14), since the spinors pick a minus sign upon $\phi_2 \to \phi_2 + 2\pi$ [16]. This is also consistent with the complex scalar Ψ having R-charge 2, see (2.26).

Therefore, allowing θ_{YM} to change by a multiple of 2π one finds the values of ϵ:

$$\epsilon = \frac{2\pi}{4N_c}k\,, \qquad k = 0, 1, \ldots, 4N_c - 1 \tag{2.32}$$

describing the anomalous breaking $\mathrm{U}(1)_R \to \mathbb{Z}_{4N_c}$.

Let us finish the section by defining the holomorphic coupling $\tau = i\frac{4\pi}{g_{YM}^2} + \frac{\theta_{YM}}{2\pi}$, which, in view of (2.24), (2.27), (2.28), and defining $u = \Lambda\rho e^{i\phi_2}$, $u_0 = \Lambda\rho_0$ is:

$$\begin{aligned} \tau &= i\frac{N_c}{\pi}\log\frac{u_0}{\Lambda}\,, \qquad \text{for } |u| \le u_0\,, \\ \tau &= i\frac{N_c}{\pi}\log\frac{u}{\Lambda}\,, \qquad \text{for } |u| \ge u_0 \end{aligned} \tag{2.33}$$

which is precisely the result expected for a ring-like distribution of vevs [16].

3. The dual with flavor

3.1 Flavor branes in the solution

The second kind of supersymmetric embeddings for D5-branes is associated to the κ-symmetry projector $\sigma_1 \Gamma_{012389}$. It corresponds to branes extended along $x_0, x_1, x_2, x_3, \sigma, \phi_1$ at any point in the other coordinates $\tilde{\theta}, \tilde{\varphi}, \rho, \phi_2$. These branes provide fundamental hypermultiplets in order to build $\mathcal{N} = 2$ SQCD. First of all, since they are extended in the σ direction, their volume is infinite (compared to the color branes), making exactly zero the effective four-dimensional gauge coupling living on them, and therefore providing a global symmetry group $\mathrm{U}(N_f)$ if they are placed on top of each other. Remember that this (and not $\mathrm{U}(N_f) \times \mathrm{U}(N_f)$) is the flavor symmetry of the $\mathcal{N} = 2$ SQCD lagrangian, due to the coupling (in $\mathcal{N} = 1$ notation)

$$Tr(\tilde{Q}\Psi Q) \tag{3.1}$$

where the $Q, \tilde{Q}$ represent the (anti)-fundamental chiral multiplets inside the $\mathcal{N} = 2$ hypermultiplet and Ψ is the chiral multiplet in the adjoint inside the $\mathcal{N} = 2$ vector multiplet. This is consistent with having only flavor D5-branes (anti D5-branes would break supersymmetry). It can be seen that the coupling (3.1) appears on the worldvolume theory of a probe color D5-brane. The easiest argument is that $\mathcal{N} = 2$ is preserved, so (3.1) is needed. Also, from the point of view of the brane intersection, it is what one expects, since Ψ parameterizes the two directions which are orthogonal to both color and flavor branes (those spanned by ρ, ϕ_2). The values of ρ, ϕ_2 at which each flavor brane is placed give the complex mass of the corresponding hypermultiplet (taking into account the identifications of section 2.2). The position in the $\tilde{\theta}, \tilde{\varphi}$ coordinates does not play any role in the low energy field theory, since we are interested in energies below the inverse radius of the S^2. However, this statement should be taken with certain criticism, since, as usually happens in the wrapped brane models, one cannot really achieve a separation of scales between the unwanted Kaluza-Klein modes and the relevant physical modes. A discussion on this point and some ideas of how to deal with this problem can be found in [22].

Let us now focus on the $\mathrm{U}(1)_R$ symmetry that, as described in section 2, corresponds to rotations of the ϕ_2 angle. Massless flavors would correspond to branes located at the origin of the moduli space $\rho = 0$, a point which is invariant under ϕ_2 rotations, and therefore preserves the $\mathrm{U}(1)_R$. A brane located at some $\rho > 0$ would correspond to a massive flavor, and, as expected, breaks explicitly the $\mathrm{U}(1)_R$.

There is an SO(3) isometry which acts on $\tilde{\theta}, \tilde{\varphi}, \phi_1$ (it is not SU(2) since ϕ_1 takes values between 0 and 2π whereas in the usual SU(2) left invariant one-forms, it would range up to 4π), which does not play a role in the low energy theory [16]. Thus, the $\mathrm{SU}(2)_R$ symmetry of the field theory cannot be entirely realized in the geometry. Nevertheless, there is another isometry in the solution, the rotations in the ϕ_1 angle, which can be identified with the $\mathrm{U}(1)_J \subset \mathrm{SU}(2)_R$. An indication that this is the case comes from comparison to a non-critical string setup. It was argued in [23] (see also [24]) that, if one considers the eight-dimensional background $\mathbb{R}_{1,3} \times \mathbb{R}_2 \times \frac{\mathrm{SL}(2,\mathbb{R})_4}{\mathrm{U}(1)}$ and places D3-branes extended along the $\mathbb{R}_{1,3}$, at the tip of the cigar, and D5-branes extended along the $\mathbb{R}_{1,3} \times \frac{\mathrm{SL}(2,\mathbb{R})_4}{\mathrm{U}(1)}$, then

		CY_2				$\mathbb{R}_2$	
	$x_{1,3}$	σ	ϕ_1	$\tilde{\theta}$	$\tilde{\varphi}$	ρ	ϕ_2
N_c D5	$-$	$\cdot$	$\cdot$	◯	◯	$\cdot$	$\cdot$
N_f D5	$-$	$-$	◯	$\cdot$	$\cdot$	$\cdot$	$\cdot$
	4D spacetime		$\mathrm{U}(1)_J \subset \mathrm{SU}(2)_R$			Energy scale	$\mathrm{U}(1)_R$
	$(-\infty, \infty)$	$[0, \infty)$	$[0, 2\pi)$	$[0, \pi]$	$[0, 2\pi)$	$[0, \infty)$	$[0, 2\pi)$

Table 1: A scheme of the setup: for the brane configuration, a line $-$ means that the brane spans a non-compact dimension, a point $\cdot$ that it is point-like in that direction and a circle ◯ that it wraps a compact cycle. The physical meaning of the different dimensions is summarized as well as their ranges. Above, it is shown which directions spanned the Calabi-Yau and which the transverse plane before backreaction.

the low energy theory living on the D3 is $\mathcal{N} = 2$ SQCD. Heuristically, one can think of this non-critical setup as the one described in this paper where the S^2 parameterized by $\tilde{\theta}, \tilde{\varphi}$ has shrunk to string scale size. Naturally, the curvature would be of the order of the string scale. In [25], it was shown that, by considering the fermions, this U(1) forms part of an SU(2) symmetry. The wrapping of the flavor D5 branes does not break this symmetry, regardless the hypermultiplet is massive or not. However, the scalars inside the hypermultiplets are charged under the $\mathrm{SU}(2)_R$, so the meson-like excitations lie in non-trivial representations of this group, as will be shown in section 4.

When two hypermultiplets have the same mass, it is possible to enter a Higgs branch by giving expectation values to Q, $\tilde{Q}$. From the gravity side, this should be described by using the non-abelian brane worldvolume action along the lines of [26].

Table 1 summarizes the setup.

3.2 Computing the flavor brane backreaction

Now that the embedding for flavor branes has been identified, the procedure of [12] can be followed in order to find a backreacted solution and the effects of having $N_f \sim N_c$ in the field theory. The unquenched solution is a solution of supergravity coupled to the flavor branes which, unlike the color branes, do not disappear into fluxes. The global flavor symmetry of the field theory corresponds to the gauge symmetry on the brane worldvolume. As explained above, this symmetry becomes global, from the four-dimensional point of view because the flavor branes have infinite volume since they are extended in the non-compact σ direction.

Clearly, the solution must depend on the masses one chooses for the fundamental hypermultiplets (the position in ρ, ϕ_2 of the branes). The case addressed below is the simplest one: each single mass breaks the $\mathrm{U}(1)_R$ symmetry but since $N_f \to \infty$, we may consider a setup in which the masses are smeared and distributed in a ring, such that the $\mathrm{U}(1)_R$ is restored. The flavor symmetry is explicitly broken to $\mathrm{U}(1)^{N_f}$. Notice the similarity with the also ring-like distribution of vevs. In particular, an infinitely thin ring at $\rho = \rho_Q$ will be considered, although the construction can be easily generalized to other configurations with the same symmetry. The choice of masses fixes the distribution of branes along the ρ, ϕ_2 directions. We still have to fix their distribution in $\tilde{\theta}, \tilde{\varphi}$. Since in

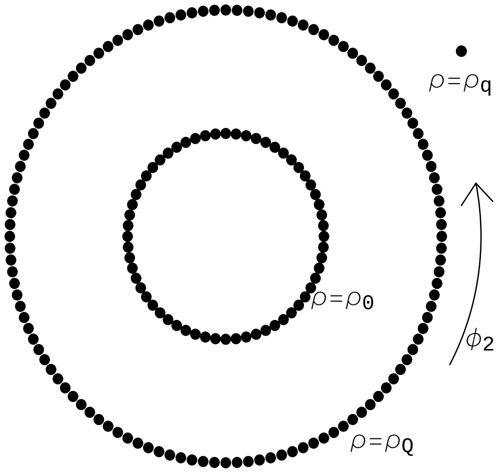

Figure 1: A scheme of the configuration on the ρ, ϕ_2 plane, which is the complex u-plane up to a factor of Λ. The inner circle at radius ρ_0 is the distribution of N_c vevs for the adjoint scalars. The outer circle represents the N_f masses of the fundamental hypermultiplets. The associated flavor branes are also extended along the transverse σ, ϕ_1 directions. These smeared distributions restore the $\mathrm{U}(1)_R$ broken by each single vev or mass. The point at a scale ρ_q represents a flavor probe brane whose dynamics will be studied in section 4.

the low energy theory these coordinates play no role, one can just consider the simplest possibility, i.e., a uniform distribution such that the SO(3) symmetry is also kept. This dramatically simplifies the task of writing an ansatz. Figure 1 depicts the situation.

The D5 flavor branes are magnetic sources for the $F_{(3)}$ RR-field strength. As in [12], a supersymmetric solution can be found by modifying the Bianchi identity for the $F_{(3)}$ and inserting the suitable ansatz in the supersymmetry variation equations (2.12). The worldvolume action for the flavor branes is:

$$S_{flavor} = T_5 \sum^{N_f} \left(- \int d^6x e^{\frac{\Phi}{2}} \sqrt{-\hat{g}_{(6)}} + \int P[C_{(6)}] \ . \right) \tag{3.2}$$

We are interested in the WZ part, which, being an infinite sum can be promoted to a ten-dimensional integral.

$$T_5 \sum^{N_f} \int P[C_{(6)}] \to T_5 \int Vol(\mathcal{Y}_4) \wedge C_{(6)} \tag{3.3}$$

where $Vol(\mathcal{Y}_4)$ is the brane density, subject to the normalization condition $\int Vol(\mathcal{Y}_4) = N_f$. For the distribution described above:

$$Vol(\mathcal{Y}_4) = \frac{N_f}{8\pi^2} \delta(\rho - \rho_Q) \ \sin\tilde{\theta} d\rho \wedge d\phi_2 \wedge d\tilde{\theta} \wedge d\tilde{\varphi} \tag{3.4}$$

The modified Bianchi identity reads[3]:

$$dF_{(3)} = 2\kappa_{(10)}^2 T_5 Vol(\mathcal{Y}_4) = g_s \alpha' \ \frac{N_f}{2} \delta(\rho - \rho_Q) \ \sin\tilde{\theta} d\rho \wedge d\phi_2 \wedge d\tilde{\theta} \wedge d\tilde{\varphi} \tag{3.5}$$

[3]In general, if for a form $F_{(n)} = dA_{(n-1)}$ there is an action $-\frac{1}{2n!} \int \sqrt{|g|} F^2 + \int G \wedge A$, the equation of

where (2.7) and (3.4) were used to obtain the second equality. The natural ansatz satisfying this condition that generalizes (2.10) is:

$$\begin{aligned} F_{(3)} &= N_c g_s \alpha' \Big[-g' d\phi_2 \wedge d\rho \wedge (d\phi_1 + \cos\tilde{\theta} d\tilde{\varphi}) - \dot{g} d\phi_2 \wedge d\sigma \wedge (d\phi_1 + \cos\tilde{\theta} d\tilde{\varphi}) + \\ &\quad + (g + \frac{N_f}{2N_c}\Theta(\rho - \rho_Q)) \sin\tilde{\theta} d\phi_2 \wedge d\tilde{\theta} \wedge d\tilde{\varphi} \Big] \ . \end{aligned} \tag{3.6}$$

where Θ is the Heaviside step function. Maintaining the same ansatz for the metric[4] (2.9) and the expressions for the Killing spinors (2.14), (2.15), one finds a very slight modification of (2.16)-(2.20), namely:

$$g + \frac{N_f}{2N_c}\Theta(\rho - \rho_Q) = -\rho z' \, , \tag{3.7}$$

$$e^{2\Phi} = \frac{\sigma}{z\dot{z}} \, , \tag{3.8}$$

$$g' = -2e^{-2\Phi}\rho\sigma\dot{\Phi} \, , \tag{3.9}$$

$$\dot{g} = -z^{-2}e^{-2\Phi}\sigma(g + \frac{N_f}{2N_c}\Theta(\rho - \rho_Q)) + 2z^{-1}\rho\sigma e^{-2\Phi}\Phi' \ . \tag{3.10}$$

As in the unflavored case, (3.10) is implied by (3.7), (3.8) and it is possible to check that these equations ensure the equation of motion for the 3-form $d(e^{\Phi} \, {}^*F_{(3)}) = 0$. The generalization of (2.20) is

$$\sigma\frac{N_f}{2N_c}\delta(\rho - \rho_Q) + \rho z(\dot{z} - \sigma\ddot{z}) = \sigma(\rho\dot{z}^2 + z' + \rho z'') \ . \tag{3.11}$$

This equation seems very difficult, if possible at all, to solve in general. However, the important point is that it uniquely defines the relevant solution. For $\rho < \rho_Q$, the solution should be equal to the unflavored one, which is explicitly known (2.21). A first argument to see this comes from an electrostatic analogy: a spherically symmetric distribution of charges does not alter the field in its interior. Such a reasoning was used in [27] in order to find the supergravity solution corresponding to shells of five-branes uniformly distributed on S^3 spheres. Notice the similarity between such a setup and the one discussed in this note. A second argument comes from field theory and holomorphic decoupling (see the final part of section 3.3).

Thus, we are left with the problem of finding the function $z(\rho, \sigma)$ for $\rho > \rho_Q$. It will be shown in section 3.3 that $z(\rho, 0)$ can be written in a simple form and leads to several

motion for the form reads $d * F = sign(g)(-1)^{D-n+1}G$. In this case, the relevant part of the action (go to string frame for this computation) reads: $-\frac{1}{2\kappa_{(10)}^2}\frac{1}{2\cdot 7!}\int\sqrt{|g|}F_{(7)}^2 + T_5\int Vol(\mathcal{Y}_4)\wedge C_{(6)}$ so the equation of motion is $\frac{1}{2\kappa_{(10)}^2} d * F_{(7)} = -T_5 Vol(\mathcal{Y}_4)$. Taking into account $F_{(3)} = - * F_{(7)}$ and we arrive at (3.5).

[4] If one considers a more general form for the metric

$$\begin{aligned} ds_{10}^2 &= g_s N_c \alpha' e^{\frac{\Phi}{2}} \Big[\frac{1}{g_s N_c \alpha'} dx_{1,3}^2 + z(d\tilde{\theta}^2 + \sin^2\tilde{\theta} d\tilde{\varphi}^2) + \\ &\quad + \alpha^2 e^{-2\Phi}(d\rho^2 + \rho^2 d\phi_2^2) + \beta^2\frac{e^{-2\Phi}}{z}\Big(d\sigma^2 + \sigma^2(d\phi_1 + \cos\tilde{\theta} d\tilde{\varphi})^2\Big)\Big] \, , \end{aligned}$$

where $\alpha(\rho,\sigma), \beta(\rho,\sigma)$, the Killing spinor equations impose that both α and β are constant.

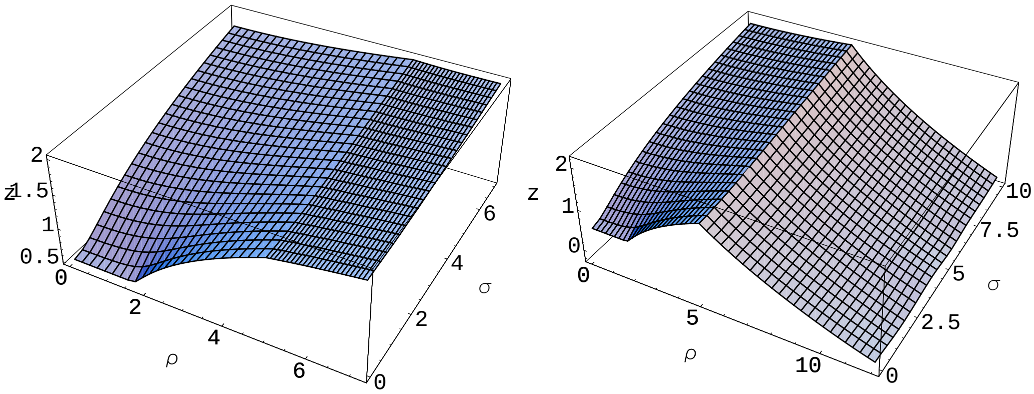

Figure 2: Numerical approximations to the function $z(\rho, \sigma)$ that enters the metric. Both plotted solutions correspond to fixing $c = 0$ ($\rho_0 = \sqrt{e}$) and $\rho_Q = 5$. It is apparent from the graphics the qualitative change of behavior at such values of ρ. On the left $N_f = N_c$. On the right, $N_f = 6N_c$, a non-asymptotically free field theory that eventually hits a Landau pole. This corresponds to z becoming zero at some value of ρ, as shown in the graph.

gauge theory predictions. By continuity, and regarding the previous paragraph $z(\rho_Q, \sigma)$ must be the same as in the unflavored case and therefore is known. From (3.7), one can read how the derivative changes when traversing the brane shell, fixing $z'(\rho_Q, \sigma)$. This set of boundary conditions fixes the solution. In figure 2, a numerical approximation to the function $z(\rho, \sigma)$ in two particular cases is presented.

There is a last subtle point that requires discussion before closing this section. We have introduced a number of fundamentals and found new equations that define a new solution. If this new solution still describes a theory with N_c colors, equation (2.5) should still hold. Let us now prove so. Comparing to (2.6), the integrals in ϕ_1, ϕ_2 are trivial, but the angle θ is not manifest in the geometry any more. This problem is solved by noticing that the components of $F_{(3)}$ relevant to this integration (those not containing $d\tilde{\theta}$ or $d\tilde{\varphi}$ can be written as (see (3.6)) $N_c g_s \alpha' dg \wedge d\phi_2 \wedge d\phi_1$ and that, at infinity, the limits of integration in θ correspond to $(\sigma = 0, \rho = \infty)$ and $(\sigma = \infty, \rho = 0)$. Then:

$$\int F_{(3)} = 4\pi^2 g_s N_c \alpha' (g|_{((\sigma=\infty,\rho=0)} - g|_{(\sigma=0,\rho=\infty)}) = 4\pi^2 g_s N_c \alpha' \tag{3.12}$$

where the equalities $g|_{(\sigma=\infty,\rho=0)} = 0$ and $g|_{(\sigma=0,\rho=\infty)} = -1$ have been used, and also the fact that g is continuous along the integration path. The first equality is trivial since for $\rho < \rho_Q$ the solution coincides with the unflavored one so g can be read from (2.22) setting $\theta = 0$. The second one will be proved in section 3.3. Thus, since (3.12) coincides with (2.6), the quantization condition still holds in the flavored case.

3.3 Gauge theory features

Let us now study the gauge theory implications of this computation. The $\rho < \rho_0$ region is unchanged with respect to section 2 when the fundamentals are present, so in the following, only the $\rho > \rho_0$ region is considered. By repeating the argument of [20] reviewed in

JHEP12(2006)032

section 2.2, it is straightforward to generalize (2.27), (2.28) to:

$$\frac{1}{g_{YM}^2} = \frac{N_c}{4\pi^2}(z|_{\sigma=0}), \tag{3.13}$$

$$\theta_{YM} = 2\Big(g|_{\sigma=0} + \frac{N_f}{2N_c}\Theta(\rho-\rho_Q)\Big)N_c\phi_2 \tag{3.14}$$

Equation (3.9) implies that $g|_{\sigma=0}$ is a constant. Repeating the argument that the solution at small ρ should be the same as the unflavored one, we find (as in (2.24)) $g|_{\sigma=0} = -1$. Thus:

$$\theta_{YM} = -2N_c\Big(1 - \frac{N_f}{2N_c}\Theta(\rho-\rho_Q)\Big)\phi_2 \tag{3.15}$$

One can compute the monodromy of the θ_{YM} angle when making a full $\mathrm{U}(1)_R$ rotation through the space of vevs (at a fixed modulus $\rho\Lambda$):

$$\phi_2 \to \phi_2 + 4\pi \quad \Rightarrow \quad \frac{\theta_{YM}}{2\pi} \to \frac{\theta_{YM}}{2\pi} - 2\Big(2N_c - N_f\Theta(\rho-\rho_Q)\Big) \tag{3.16}$$

which depends on $N_f\Theta(\rho-\rho_Q)$, the number of fundamental hypermultiplet masses encircled by the path. This result is in exact agreement with the field theory expectations (for a review, see [28]).

Let us now turn to g_{YM}. Using $g|_{\sigma=0} = -1$, it is immediate to integrate (3.7) at $\sigma = 0$ in order to find the Yang-Mills coupling (3.13) Imposing that z matches the unflavored solution for $\rho < \rho_Q$ and is continuous at $\rho = \rho_Q$, one obtains:

$$\frac{1}{g_{YM}^2} = \frac{1}{4\pi^2}\Big[\Big(N_c - \frac{N_f}{2}\Theta(\rho-\rho_Q)\Big)\log\rho + \frac{N_f}{2}\Theta(\rho-\rho_Q)\log\rho_Q\Big] \tag{3.17}$$

Using (2.29), the β-function at an energy scale μ is straightforwardly computed: $\beta(g_{YM})(\mu) = -\frac{g_{YM}^3}{8\pi^2}(N_c - \frac{N_f(\mu)}{2})$ where $N_f(\mu)$ is defined as the number of flavors for which the modulus of their masses is smaller than the scale. The fact that matter fields with bigger mass do not contribute is usually an approximate statement, but, due to the radial symmetry, in this case it is exact as will be shown below. Their effect is just to modify the dynamically generated scale Λ. For $N_f \geq 2N_c$, the theory loses asymptotic freedom and for $N_f > 2N_c$, it eventually hits a Landau pole. In the gravity solution, this is translated in the fact that when $N_f > 2N_c$, the function z vanishes at some finite ρ and the metric becomes singular. An analogous behavior in a D3D7 solution was discussed in [11].

The holomorphic coupling can be written by using (3.15) and (3.17) (define $u_Q = \rho_Q\Lambda$):

$$\begin{aligned}
\tau &= i\frac{N_c}{\pi}\log\frac{u_0}{\Lambda}, \qquad \text{for } |u| \leq u_0, \\
\tau &= i\frac{N_c}{\pi}\log\frac{u}{\Lambda}, \qquad \text{for } u_0 \leq |u| \leq u_Q, \\
\tau &= i\frac{N_f}{2\pi}\log\frac{u_Q}{\Lambda} + \frac{i}{\pi}(N_c - \frac{N_f}{2})\log\frac{u}{\Lambda}, \qquad \text{for } |u| \geq u_Q,
\end{aligned} \tag{3.18}$$

This behavior precisely matches the field theory result. The proof is a simple generalization of the one of [16] for the unflavored case. The coupling is given in terms of the so-called prepotential. As argued in [29], non-perturbative contributions to the prepotential vanish in the large N limit as long as one probes the theory far enough from the vevs $|u - a_i| \gg N_c^{-1}$ (or, in this case, also from the masses of the hypermultiplets). The perturbative prepotential can be read, for instance, from [30]. Slightly adapting conventions in order to match those of [16]:

$$\mathcal{F} = \frac{i}{8\pi}\sum_{i\neq j}(a_i - a_j)^2 \log\frac{(a_i - a_j)^2}{\Lambda^2} - \frac{i}{8\pi}\sum_{\alpha=1}^{N_f}\sum_{i=1}^{N_c}(a_i - m_\alpha)^2 \log\frac{(a_i - m_\alpha)^2}{\Lambda^2} \tag{3.19}$$

The gauge group is broken to $\mathrm{U}(1)^{N_c}$ due to the vevs of the scalars. We now probe the theory by considering all but one of the vevs in the ring-like distribution and moving around the other one, u, so we have $\mathrm{U}(1)^{N_c-1}\times\mathrm{U}(1)$ where the last $\mathrm{U}(1)$ corresponds to the probe brane. In the large N_c limit, the coupling is:

$$\tau(u) = \frac{\partial^2\mathcal{F}}{\partial u^2} = \frac{i}{2\pi}\sum_{i=1}^{N_c}\log\frac{(u-a_i)^2}{\Lambda^2} - \frac{i}{4\pi}\sum_{\alpha=1}^{N_f}\log\frac{(u-m_\alpha)^2}{\Lambda^2} \tag{3.20}$$

In the large N_c, N_f limit, the sum over discrete distributions can be replaced by integrals, namely:

$$\tau(u) = \frac{i}{2\pi}\int d^2a\rho_{<\Psi>}(a)\log\frac{(u-a)^2}{\Lambda^2} - \frac{i}{4\pi}\int d^2m\rho_{m_Q}(m)\log\frac{(u-m)^2}{\Lambda^2} \tag{3.21}$$

where $\rho_{<\Psi>}(a)$, $\rho_{m_Q}(m)$ are the density distributions of vevs for the adjoints and masses for the fundamentals respectively. The densities must satisfy the normalization $\int d^2a\rho_{<\Psi>}(a) = N_c$, $\int d^2m\rho_{m_Q}(m) = N_f$. Therefore, the ring-like distributions are $\rho_{<\Psi>}(a) = \frac{N_c}{2\pi u_0}\delta(|a| - u_0)$ and $\rho_{m_Q}(m) = \frac{N_f}{2\pi u_Q}\delta(|m| - u_Q)$. Performing the integrals (3.21) and adjusting an additive constant that can be reabsorbed as a rescaling of Λ one finds precisely (3.18). The rescaling of Λ is what one usually obtains from holomorphic decoupling.

In the context of fractional branes at orbifolds with massless fundamental hypermultiplets, results similar to those reported in this section were presented in [3, 8].

4. Mesonic excitations

This section is devoted to the analysis of the meson-like excitations, both in the quenched and unquenched backgrounds. In order to address this question, let us add an additional flavor to the theory with a mass defined by its position in ρ, ϕ_2. The flavor group is now $\mathrm{U}(1)^{N_f+1}$. Obviously, the effect in the geometry of this new brane is suppressed by a power N^{-1} and is negligible. Thus, its excitations can be described by considering it a probe in the background described in the previous sections.

The spectrum can be computed by expanding the action to quadratic order in the fluctuations. The analysis is similar to that in [6], where also an $\mathcal{N} = 2$ theory was analyzed. There are, however, important differences in the low energy field theory: apart from having a large number of hypermultiplets, here there is no massless adjoint hypermultiplet.

JHEP12(2006)032

Let us consider a probe brane extended along $x_0, x_1, x_2, x_3, \sigma, \phi_1$ and sitting at a point defined by $\tilde{\theta}_q, \tilde{\varphi}_q, \rho_q, \phi_{2q}$. Its worldvolume action reads (below, string frame will be used, so the metric (2.9) should be multiplied by $e^{\frac{\Phi}{2}}$):

$$S_{D5} = T_5 \left(-\int d^6\xi e^{-\Phi} \sqrt{-P[g] + 2\pi\alpha' F} + \int P[C_{(6)}] + \frac{(2\pi\alpha')^2}{2} \int P[C_{(2)}] \wedge F \wedge F \right) \tag{4.1}$$

although the last term does not contribute at quadratic order, as can be seen by choosing a gauge in which $C_{(2)} = g_s N_c \alpha' \left(g + \frac{N_f}{2N_c} \Theta(\rho - \rho_Q) \right) d\phi_2 \wedge (d\phi_1 + \cos\tilde{\theta} d\tilde{\varphi})$. We need the expression for the $C_{(6)}$ in the background which is defined as $dC_{(6)} = F_{(7)} = - * F_{(3)}$. Taking the Hodge dual of (3.6), one finds:

$$F_{(7)} = g_s N_c \alpha' dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge \Big[- \frac{g' e^{4\Phi} z}{\rho\sigma} \sin\tilde{\theta} d\tilde{\theta} \wedge d\tilde{\varphi} \wedge d\sigma + \\ + \frac{\dot{g} z^2 e^{4\Phi}}{\rho\sigma} \sin\tilde{\theta} d\tilde{\theta} \wedge d\tilde{\varphi} \wedge d\rho - \big(g + \frac{N_f}{2N_c} \Theta(\rho - \rho_Q) \big) \frac{\sigma}{\rho z^2} d\rho \wedge d\sigma \wedge (d\phi_1 + \cos\tilde{\theta} d\tilde{\varphi}) \Big] \tag{4.2}$$

Using (3.7)-(3.10), one can write the following simple expression for the RR-potential:

$$C_{(6)} = g_s N_c \alpha' dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge \left(z e^{2\Phi} \sin\tilde{\theta} d\tilde{\theta} \wedge d\tilde{\varphi} - \frac{\sigma}{z} d\sigma \wedge (d\phi_1 + \cos\tilde{\theta} d\tilde{\varphi}) \right) \tag{4.3}$$

4.1 Fluctuation of the scalar fields

In order to describe the small fluctuations, the simplest method is to consider the static gauge and use x_μ, σ, ϕ_1 as worldvolume coordinates. The embedding can be written as:

$$\tilde{\theta} = \tilde{\theta}_q + \delta\tilde{\theta} , \quad \tilde{\varphi} = \tilde{\varphi}_q + \delta\tilde{\varphi} , \quad \rho = \rho_q + \delta\rho , \quad \phi_2 = \phi_{2q} + \delta\phi_2 , \tag{4.4}$$

where all the fluctuations depend on x_μ, σ, ϕ_1. It is immediate to see that at quadratic order, these scalar fluctuations do not mix with those of the worldvolume gauge field which will be studied in the next section. Expanding the action (4.1) up to quadratic order one finds:

$$\begin{aligned} S = & \; T_5 g_s N_c \alpha' \int d^4x d\phi_1 d\sigma \Big[\frac{1}{2} \frac{\sigma}{z} e^{-2\Phi} g_s N_c \alpha' (\partial_x \delta\rho)^2 + \frac{1}{2} \sigma (\partial_\sigma \delta\rho)^2 + \frac{1}{2\sigma} (\partial_{\phi_1} \delta\rho)^2 + \\ & + \frac{1}{2} \rho_q^2 \frac{\sigma}{z} e^{-2\Phi} g_s N_c \alpha' (\partial_x \delta\phi_2)^2 + \frac{1}{2} \rho_q^2 \sigma (\partial_\sigma \delta\phi_2)^2 + \frac{1}{2\sigma} \rho_q^2 (\partial_{\phi_1} \delta\phi_2)^2 + \\ & + \frac{1}{2} g_s N_c \alpha' \sigma (\partial_x \delta\tilde{\theta})^2 + \frac{1}{2} \sigma z e^{2\Phi} (\partial_\sigma \delta\tilde{\theta})^2 + \frac{z e^{2\Phi}}{2\sigma} (\partial_{\phi_1} \delta\tilde{\theta})^2 + \\ & + \frac{1}{2} g_s N_c \alpha' \sigma \sin^2\tilde{\theta}_q (\partial_x \delta\tilde{\varphi})^2 + \frac{1}{2} \sigma z e^{2\Phi} \sin^2\tilde{\theta}_q (\partial_\sigma \delta\tilde{\varphi})^2 + \frac{z e^{2\Phi}}{2\sigma} \sin^2\tilde{\theta}_q (\partial_{\phi_1} \delta\tilde{\varphi})^2 + \\ & + z e^{2\Phi} \sin\tilde{\theta}_q \left((\partial_\sigma \delta\tilde{\theta})(\partial_{\phi_1} \delta\tilde{\varphi}) - (\partial_{\phi_1} \delta\tilde{\theta})(\partial_\sigma \delta\tilde{\varphi}) \right) \Big] \end{aligned} \tag{4.5}$$

where z, Φ should be understood as functions of σ at $\rho = \rho_q$. We have defined $(\partial_x \delta\rho)^2 = \eta^{\mu\nu} (\partial_\mu \delta\rho)(\partial_\nu \delta\rho)$ where μ, ν span the four Minkowski directions. The fluctuations of $\delta\rho, \delta\phi_2$ are already decoupled and it is very easy to decouple those for $\delta\tilde{\theta}, \delta\tilde{\varphi}$. Let us expand the fields as:

$$\begin{aligned}
\delta\rho &= \chi_1(\sigma)e^{ik\cdot x}\cos(l\phi_1)\ , \\
\delta\phi_2 &= \chi_2(\sigma)e^{ik\cdot x}\cos(l\phi_1)\ , \\
\delta\tilde{\theta} &= \frac{\chi_+(\sigma)+\chi_-(\sigma)}{2}\sin\tilde{\theta}_q e^{ik\cdot x}\sin(l\phi_1)\ , \\
\delta\tilde{\varphi} &= \frac{\chi_+(\sigma)-\chi_-(\sigma)}{2}e^{ik\cdot x}\cos(l\phi_1)\ .
\end{aligned} \tag{4.6}$$

The equation of motion for χ_1, χ_2 reads:

$$\frac{\sigma}{ze^{2\Phi}}\bar{M}^2\chi_i + \partial_\sigma(\sigma\partial_\sigma\chi_i) - \frac{l^2}{\sigma}\chi_i = 0\ . \tag{4.7}$$

where we have defined the four-dimensional mass $M^2 = -k_\mu k^\mu$ and $\bar{M}^2 = g_s N_c \alpha' M^2$.

The equations for $\chi_\pm$ read:

$$\bar{M}^2\chi_\pm + \frac{1}{\sigma}\partial_\sigma(\sigma z e^{2\Phi}\partial_\sigma\chi_\pm) - \frac{l^2 z e^{2\Phi}}{\sigma^2}\chi_\pm \mp \frac{l}{\sigma}\partial_\sigma(ze^{2\Phi})\chi_\pm = 0 \tag{4.8}$$

4.2 Fluctuation of the gauge field

The equation of motion for the gauge field at linear order is:

$$\partial_a(e^{-\Phi}\sqrt{-\det(P[g])}F^{ab}) = 0 \tag{4.9}$$

where a, b run over the six worldvolume coordinates and, again, μ, ν over the Minkowski four-space and are contracted with the flat metric $\eta_{\mu\nu}$. Let us now choose a Lorentz gauge $\partial_\mu A^\mu = 0$. The following relation is found:

$$\partial_{\phi_1}A_{\phi_1} = -\sigma\partial_\sigma(\sigma A_\sigma) \tag{4.10}$$

Defining a transverse polarization tensor ϵ_μ and expanding:

$$\begin{aligned}
A_\mu &= \epsilon_\mu\chi_3(\sigma)e^{ik\cdot x}\cos(l\phi_1)\ , \\
A_\sigma &= \frac{1}{\sigma}\chi_4(\sigma)e^{ik\cdot x}\cos(l\phi_1)\ .
\end{aligned} \tag{4.11}$$

one finds from (4.9) that χ_3, χ_4 also satisfy equation (4.7).

4.3 Analysis of the spectrum

It is interesting to notice certain similarity of the equations (4.7), (4.8) with (3.6) and (3.31) of [6], respectively. Thus, the analysis of the spectrum is also similar to the one in [6].

However, the l we have used here is not the eigenvalue of some J^2 operator acting on the $\mathrm{SU}(2)_R$ generators, but it is the charge under a $\mathrm{U}(1)_J \subset \mathrm{SU}(2)_R$, i.e., the eigenvalue of a J_3 operator. The relation between the two would require a better understanding. The bosonic content of the generic ($l \geq 2$) $\mathcal{N} = 2$ massive vector multiplet is given by a massive vector and three real scalars in the $\frac{l}{2}$ of $\mathrm{SU}(2)_R$ and two real scalars in the $\frac{l-2}{2}$ and $\frac{l+2}{2}$.

JHEP12(2006)032

Since (4.7) is satisfied by χ_i, $i = 1, 2, 3, 4$, we have in fact a massive vector and three real scalars with the same mass in the same representation of $\mathrm{SU}(2)_R$. In order to complete the multiplet, two more real scalars with the same mass are needed. They are the $\chi_\pm$ of (4.8). This can be shown by mapping equation (4.7) to (4.8), following a procedure used in [31] for a similar case. In fact:

$$\begin{aligned}\chi_{+,(l=L)} &= \sigma^{-L-1}\partial_\sigma(\chi_{i,(l=L+1)}\sigma^{L+1}) \\ \chi_{-,(l=L)} &= \sigma^{L-1}\partial_\sigma(\chi_{i,(l=L-1)}\sigma^{1-L})\end{aligned} \tag{4.12}$$

In order to prove this, substitute in (4.7) $l \to L \pm 1$ and define $F_\pm = \sigma^{1\pm L}\chi_i$. Multiply the equation by $ze^{2\Phi}\sigma^{\pm L}$ and derive with respect to σ. Now, substituting $\partial_\sigma F_\pm = \chi_\pm \sigma^{1\pm L}$ and multiplying by $\sigma^{-1\mp L}$, one arrives at (4.8) with $l = L$.

Thus, the bosonic fluctuations yield the bosonic content of a tower of $\mathcal{N} = 2$ massive vector multiplets which can be identified with the mesonic excitations of the theory. The fermionic spectrum can be inferred from supersymmetry or could be directly computed along the lines of [32].

The physical, mesonic, excitations should be those satisfying equations (4.7) or (4.8) and moreover being regular at $\sigma = 0$ and normalizable as $\sigma \to \infty$. Normalizability is a problematic issue in this setup due to ill-behaved UV and will be discussed in section 4.4.

About regularity at the origin, one would need that the χ functions are finite and even (odd) around $\sigma = 0$ when l is even (odd). From (4.7), one sees that near $\sigma = 0$, $\chi_{i,l} = c_1\sigma^l + c_2\sigma^{-l}$. Clearly, regularity selects $c_2 = 0$ and $\chi_{i,l} \propto \sigma^l$ which in fact satisfies the criterion of parity stated above. For the χ_+ modes, substituting in (4.12), we find $\chi_{+,l} \propto \sigma^l$ which is also fine with the parity condition. Checking the χ_- modes, involves an extra subtlety since substituting the leading χ_i behavior in (4.12) one just gets zero. But from (4.7), noting that near $\sigma = 0$, $\dot{z} \propto \sigma$ we see that both z and $e^{2\Phi}$ behave as $k_1 + k_2\sigma^2$ where k_1, k_2 are some constants. Thus, the subleading behavior of χ_i is described by $c_1\sigma^l(1 + k_3\sigma^2)$, so substituting in (4.12) one also finds $\chi_{-,l} \propto \sigma^l$ which, again, is fine.

4.4 Estimate of the mass spectrum

In D5-brane solutions, like the one we are dealing with or the Maldacena-Núñez background, the dilaton diverges in the UV. The supergravity formalism cannot be trusted in that region, and the UV completion of the field theory is a little string theory. Therefore, it is not a surprise that there are problems in the UV when trying to determine the spectrum. In fact, there is no normalizable mode. However, physically, one would expect to have some modification of the UV that cures this problem, since in the dual theory there are, of course, physical excitations. In this section, a regularization procedure similar to those in [33, 34] is proposed, which basically consists of cutting off the ill-behaved region. It is curious to notice the qualitative difference with [35], where glueballs in Maldacena-Núñez were considered. In that case, even though there was no normalizable mode, the authors found that there existed one leading and one subleading mode in the UV and identified the subleading as the physical one (in a different context, the same criterion was also proposed in [36]). In the present case, there is no leading UV mode because the two modes behave as sine and cosine of some function of σ.

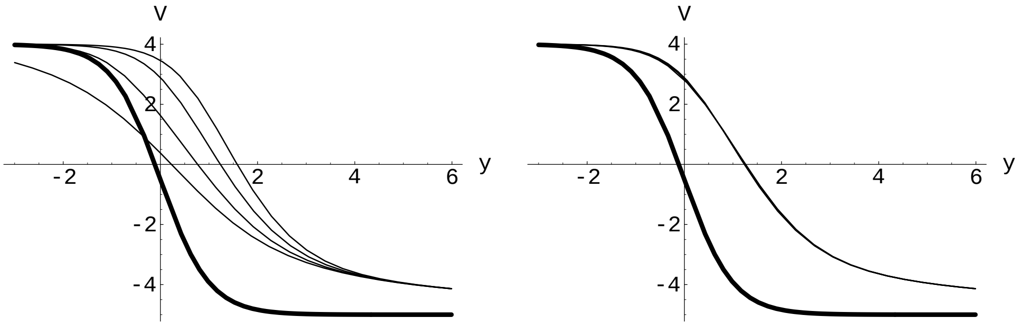

Figure 3: The figure in the left represents the potential (4.15) with fixed $\frac{N_f}{N_c}=0,\ l=2,\ \bar{M}=3,\ c=-1$ for different values of $\rho_q=1,2,3,4$ (the biggest the ρ_q the upper is the line). The thick line corresponds to (4.16). The figure on the right is for fixed $\frac{N_f}{N_c}=0,\ l=2,\ \bar{M}=3,\ \rho_q=3$ for $c=-1,0,1$. The different lines are almost coincident and the meson masses should not strongly depend on c.

In order to justify the procedure, it is interesting to notice the similarity in the UV of this setup to the D5D5 brane intersection in flat space. Of course, in such a case, there are also no normalizable modes [31, 37]. But in the UV, by a chain of dualities one can connect the system to an M5M5 intersection which indeed has physical normalizable oscillations [37]. Let us define:

$$y = \log \sigma \tag{4.13}$$

such that equation (4.7) can be written in the Schrodinger form:

$$\frac{d^2\chi_i}{dy^2} - V(y)\chi_i = 0 \ . \tag{4.14}$$

The potential reads:

$$V(y) = l^2 - \bar{M}^2 \frac{e^{2y}}{ze^{2\Phi}} \tag{4.15}$$

where $y \in (-\infty, \infty)$. The function $ze^{2\Phi}$ is implicitly known in the quenched case (see section 2) and can be determined numerically in the unquenched case, as described in 3.2.

For D5D5 intersecting in flat space [37] the potential reads:

$$V(y) = l^2 - \bar{M}^2 \frac{e^{2y}}{e^{2y}+1} \tag{4.16}$$

Figure 3 shows the qualitative similarity between the two cases. For simplicity, the potential (4.15) is plotted for the quenched case, although the qualitative behavior for the unquenched one is the same. From the figure, we see that starting from $y \approx 4 \Rightarrow \sigma \approx 50$, all the potentials coincide, i.e., the IR effects are no longer important. It is natural to choose such a scale as a cutoff. In the plot, the UV problem is apparent since $V(y\to\infty) < 0$ and such a Schrödinger potential cannot have a discrete spectrum.

The masses can be estimated by applying the WKB method, which should be reliable for large excitation number. Let us focus on equation (4.7). In the notation of appendix A, we have:

$$
\begin{aligned}
f = \sigma &\Rightarrow f_1 = f_2 = s_1 = r_1 = 1 \ , \\
h = \frac{\sigma}{ze^{2\Phi}} &\Rightarrow s_2 = 1 \ , \quad r_2 = -1 \ , \\
p = -\frac{l^2}{\sigma} &\Rightarrow p_1 = p_2 = -l^2 \ , \quad s_3 = r_3 = -1
\end{aligned}
\tag{4.17}
$$

where it was used that $ze^{2\Phi} \sim \sigma^2$ (up to logarithms) for large σ. In fact, the problem for large σ appears in this formalism due to the fact that $r_1 - r_2 - 2$ is zero, but it should be a positive number in order to apply the WKB. Now, suppose that we can change the UV by that of the M5M5 intersection, which would have $r_2 = -2$ [37] and leave all the rest unchanged. With this, the WKB approximation to the masses reads (A.5):

$$
\bar{M}^2_{WKB} \approx \frac{\pi^2}{\xi^2} n(n-1+3l) \, , \qquad n \geq 1 \tag{4.18}
$$

where (A.6):

$$
\xi = \int d\sigma \sqrt{\frac{h}{f}} = \int_0^{\sigma_{cutoff}} d\sigma (ze^{2\Phi})^{-\frac{1}{2}} \tag{4.19}
$$

Now, the problem for large σ appears in the fact that this integral is divergent if $\sigma_{cutoff} \to \infty$. As said above, a natural scale to cut the integral is around $y \approx 4 \Rightarrow \sigma \approx 50$. Although this involves some arbitrariness, everything is fixed in terms of a single parameter σ_{cutoff} and thus, it should be possible to estimate how the meson masses depend on the physical parameters c, ρ_q, N_f, ρ_Q.

Figure 4 shows such a dependence in the quenched case, which is easier to deal with since there is an expression for z (2.21) and no numerical integration of a system of PDEs is needed. The quantity ξ^{-1}, proportional to the meson mass, is plotted versus ρ_q. The graph on the left shows how changing σ_{cutoff} does not change the qualitative properties of the graph, although of course shifts ξ. The graph on the right, shows the behavior for different values of c (related to ρ_0 by (2.23)). For $\rho_q \gg \rho_0$, the curves become degenerate as expected on general grounds. The minimal value for the meson masses is attained at $\rho_q = \rho_0$. It is natural to conjecture that this is due to the fact that when $< \Psi >= m_q$ for some of the eigenvalues, there are cancellations between the superpotential term (3.1) and the mass term yielding effectively massless quarks.

An important question is how the quantum effects produced by unquenched flavor affect the spectrum of physical excitations. From the discussion of section 3.2, it is clear that for $\rho_q \leq \rho_Q$ the spectrum is unchanged by the unquenched fundamentals. When $\rho_q > \rho_Q$, the computation requires a delicate numerical analysis of the PDE (3.11) in order to obtain the function $ze^{2\Phi}(\sigma)$ for given $(\frac{N_f}{N_c}, \rho_Q, \rho_q, c)$. I could not find any significative change of the meson masses when the $N_f \sim N_c$ hypers are introduced, even when $\rho_q > \rho_Q$. However, this conclusion should be taken with a grain of salt due to the difficulty of obtaining a precise numerical solution near $\sigma = 0$ and also for $\rho_q \gg \rho_Q$.

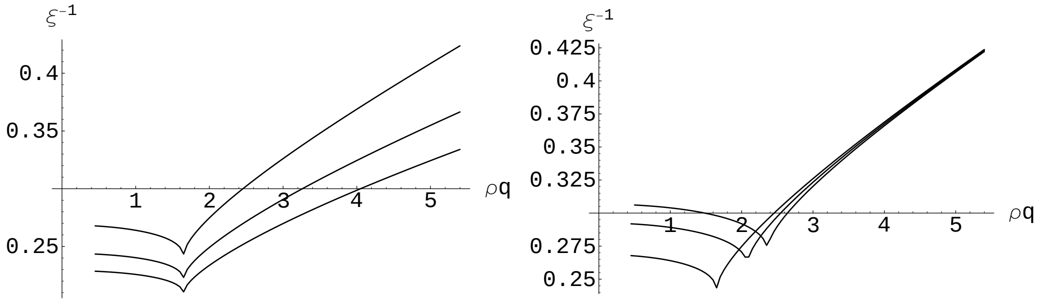

Figure 4: On the left, ξ^{-1} versus ρ_q for $c = 0$. Starting from above, the three lines correspond to $\sigma_{cutoff} = 50, 70, 90$. On the right, ξ^{-1} versus ρ_q with fixed $\sigma_{cutoff} = 50$ for $c = 0, 2, 4$. In both plots, $\frac{N_f}{N_c} \to 0$.

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A. WKB approximation, useful formulae

This appendix collects the results and notation of [38] concerning the WKB approximation. They have been used in section 4.4. Consider a differential equation:

$$\partial_\sigma(f(\sigma)\partial_\sigma\phi) + (M^2 h(\sigma) + p(\sigma))\phi = 0 \tag{A.1}$$

where the functions f,h,p behave near $\sigma \to 0$, $\sigma \to \infty$ as:

$$\begin{aligned} f &\approx f_1\sigma^{s_1}\,, \qquad h \approx h_1\sigma^{s_2}\,, \qquad p \approx p_1\sigma^{s_3}\,, \qquad (\sigma \to 0) \\ f &\approx f_2\sigma^{r_1}\,, \qquad h \approx h_2\sigma^{r_2}\,, \qquad p \approx p_2\sigma^{r_3}\,, \qquad (\sigma \to \infty). \end{aligned} \tag{A.2}$$

The WKB approximation is consistent provided $s_2 - s_1 + 2$; $r_1 - r_2 - 2$ are positive numbers and $s_3 - s_1 + 2$; $r_1 - r_3 - 2$ are positive or zero. Define:

$$\alpha_1 = s_2 - s_1 + 2\,, \qquad \beta_1 = r_1 - r_2 - 2 \tag{A.3}$$

and

$$\begin{aligned} \alpha_2 &= |s_1 - 1| \qquad \text{or} \qquad \alpha_2 = \sqrt{(s_1 - 1)^2 - 4\frac{p_1}{f_1}} \qquad (\text{if} \quad s_3 - s_1 + 2 = 0) \\ \beta_2 &= |r_1 - 1| \qquad \text{or} \qquad \beta_2 = \sqrt{(r_1 - 1)^2 - 4\frac{p_2}{f_2}} \qquad (\text{if} \quad r_1 - r_3 - 2 = 0) \end{aligned} \tag{A.4}$$

The masses are approximated by:

$$M^2 = \frac{\pi^2}{\xi^2} n(n - 1 + \frac{\alpha_2}{\alpha_1} + \frac{\beta_2}{\beta_1}) + \mathcal{O}(n^0)\,, \quad n \geq 1 \tag{A.5}$$

with:

$$\xi = \int_0^\infty d\sigma \sqrt{\frac{h}{f}} \tag{A.6}$$

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